A quick refresher course on sea waves

needed to understand the tools of Coastal Management
AY 2016-2017

Paragraphs in italics provide exercises to be carried out in the classroom or as homework

Vincent van Gogh, View of the Sea, Scheveningen, 1882 https://www.vangoghmuseum.nl/en/collection/s0416M1990

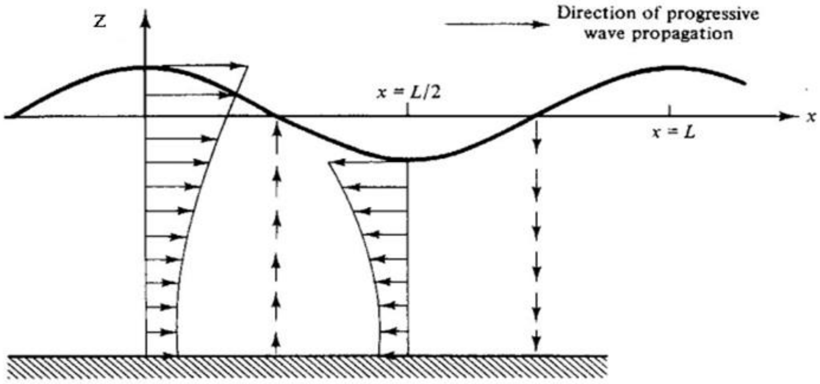
Airy / Stokes I

A simplified model of water waves is provided by Airy / Stokes I theory

$$\eta = \frac{H}{2}\cos(kx - \sigma t)$$

Also sometime called " a sine wave" since the instantaneous water height η(x,t) is a sine function of space x and time t[1].

k:wave number; K= 2π/L; L: wavelength ; σ frequency; σ = 2π/T; T :period



(from Dean e Dalrymple, 1991)

It can be shown that the whole "wave train" moves with velocity C=L/T.
We therefore define
C=L/T "Wave speed" (also: " celerity")

[1] Obviously a sine function is simply a shifted cosine

Please note that this is the velocity of the wave system, NOT the velocity of water particles- which will be considered in the following

A very important remark: σ=2π/T e di k=2π/L (and therefore T and L) are bound to each other according to the equation

$$L = \frac{g}{2\pi} T^2 \tanh \frac{2\pi h}{L} \qquad 3\,*$$

(h being the water depth)

This very important formula (“dispersion equation”) can be written in many ways: e.g.

$$T = \sqrt{\frac{\frac{2\pi}{g} L}{\tanh \frac{2\pi h}{L}}} \qquad 3**$$

Or , else:

$$\sigma^2 = gk\, tanh(\, kh\,) \qquad 3***$$

As stated above a sine wave train will move over a distance L in time T

From the previous, we also have the following

$$C = \frac{L}{T} = \frac{g}{2\pi} T tanh(hk) \qquad 3****$$

(you don’t have to learn all these formulas!: just one form, and then you can derive the others)

It useful to remember the asymptotic values for deep water and shallow water:

C0 is the wave speed on deep water (h>L) and is given by

$$Co = \frac{Lo}{T} = \frac{2\pi}{g} T \qquad 4$$

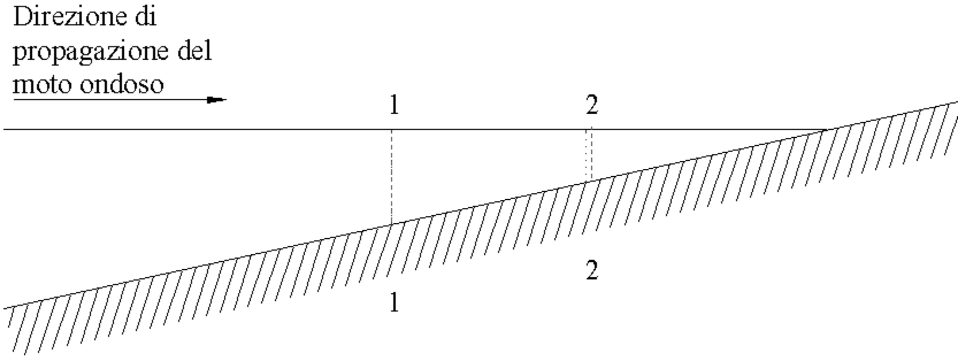
While in shallow water (h<L/10)

$$Cs = \sqrt{gh} \qquad 4*$$

Shoaling

As a wave train approaches the coast, it undergoes some transformation due to the effect of the water depth h.

We shall assume a mild slope of the bottom (basically: a beach), with a linear and parallel bathymetry.



as the wave moves from off-shore to shore (1 towards 2 in the picture) there is no change of period T2, while L and C change according to:

$C = \frac{L}{T} = \frac{g}{2\pi} T tanh(hk)$ 3****

And

$$L = \frac{g}{2\pi} T^2 tanh(hk)$$

And in deep water („Asymptotic", easy to calculate from 4 and 4*)

$Lo = \frac{g}{2\pi} To^2$ 5

$Co = \frac{Lo}{T} = \frac{2\pi}{g} T$ 5*

There is a technical difficulty in dealing with the equations above: they cannot be solved for L since this unknown appears on both sides oft he equation.

There are various ways to tackle with this problems. For our purposes it suffices to know that there are various numerical approximations: One of them is Hunt's formula, which is already coded in the EXCEL files provided in the exercises.

The height H of a wave depends by the local depths h;

H = Ho Ks 6

Ks is the shoaling coefficient:

$$Ks = H / Ho = \sqrt{\frac{C_{g0}}{Cg}} = \sqrt{\frac{C_0}{C}\frac{0.5}{n}}$$ (NO NEED TO LEARN BY HEART) 7

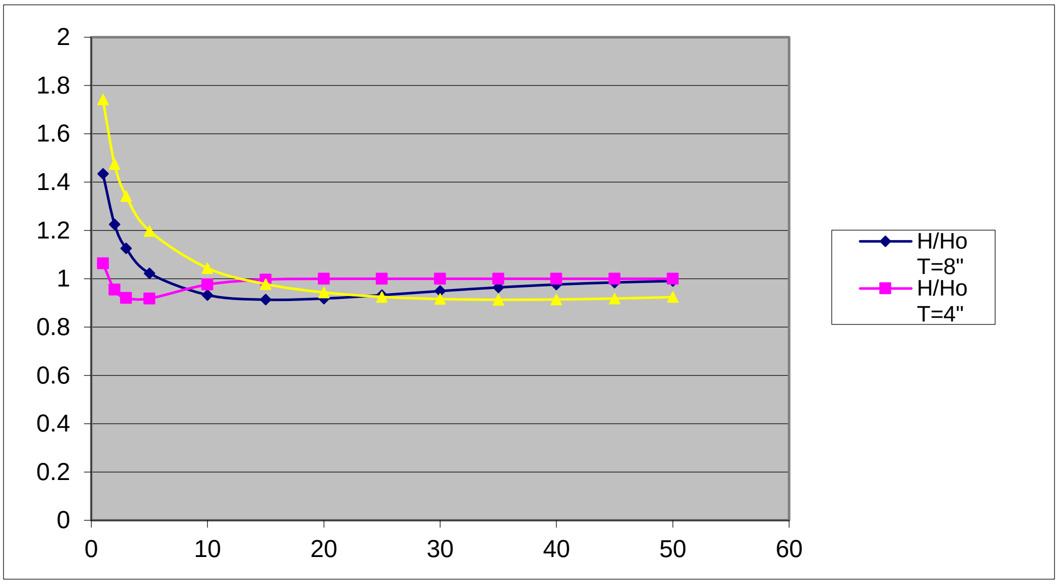
Cg is called "Group Velocity and is given by Cg=n C;

And n is given by

[2] The slope is mild, it can be therefore be assumed that the formulas above 1-5 hold even if tbe bottom is not horizontal

$$n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right)$$ (NO NEED TO LEARN BY HEART) 8

The following diagram show the behaviour of Ks as a function of depth for a wave train that approaches the coast.



shoaling coefficient K w.r.t. depth h for various values of wave period T

Here too the "asymptotic" solutions for deep water (L>h) and shallow water (L<<h) are very useful, and easy to remember

in deep water $\boldsymbol{n = 0,5 \quad C_g = 0,5C}$

in shallow water $\boldsymbol{C = \sqrt{gh} \qquad n = 1 \quad C_g = Cn = \sqrt{gh}}$

Ks links the wave height H at the local depth to offshore H_0 (deep water).

~~Exercise: Using Hunt formula to show that as the water depths decreases, C and L increase.~~

Compute C , L Ks on shallow water (you do not need the Hunt's formula there)- Se exercises

ENERGY

It can be shown that the energy of a single sine wave is given by

$$Ew = \left(\frac{1}{8}\rho g H^2\right)$$

A sine wave train carries along an energy flux given by

$$\overline{F} = \left(\frac{1}{8} \rho g H^2 \right) \cdot C_g = E \cdot C_g$$ IMPORTANT!

(In Watts, if all the measures are in IS)

With, as above

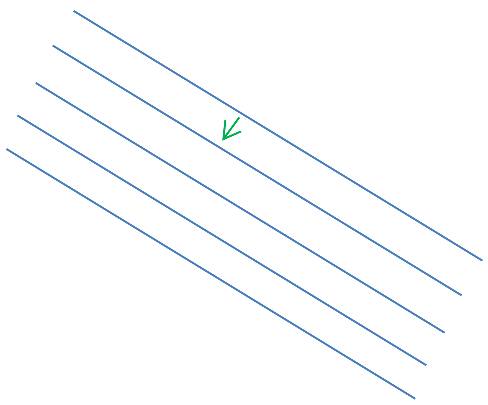
$$C_g = C \cdot n$$

and

$$n = \frac{C_g}{C} = \frac{1}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right)$$ NO NEED TO MEMORIZE

REFRACTION

If an Airy/Stokes I sine wave train moves over the sea surface, it can be visualized as follows.



The parallel lines represent points of equal phases e.g. the crests, or the troughs.

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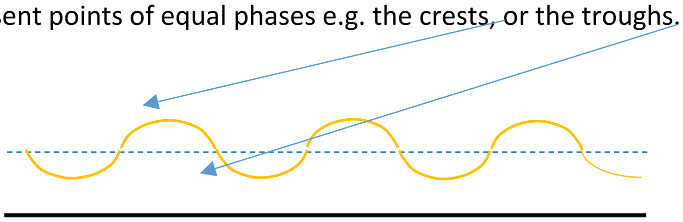
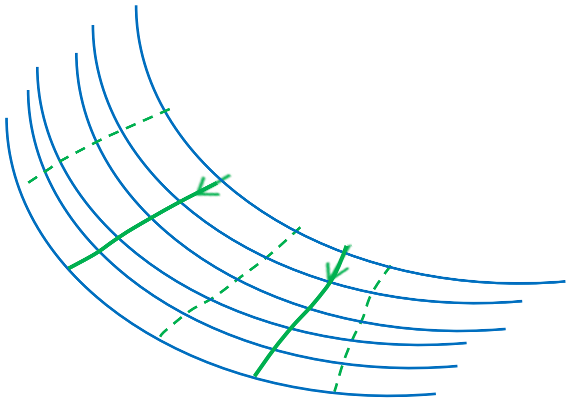


They are also called wave fronts. Perpendiculars to the wave fronts are called “rays” and the vectors along the rays are called “propagation vectors” (green arrow on the picture above)

If the wave fronts are rectilinear, the theory and the formulas described above apply without any change. The rays are straight lines, and the propagation vectors are all aligned.

The wave fronts can also be curves (for reasons that will be explained in the following); if the curvature is not too high (local curvature radius >>L) the same results apply along the rays (green thick lines)



It is therefore acceptable to assume that along the propagation (rays) the wave behaves nearly as the one-dimensional wave described above

When the waves move over a varying bathymetry (depth profile) the wave velocity changes as a function of h; but h(x,y) is generally different along the wave front, so it will change its shape and the rays will rotate. This effect is called “rifraction”

A problem which often arises in connection with the coastal sand regime, is how to calculate the angle of arrival of a wave train on the beach, once its direction offshore is known

However, conceptually at least, the problem can be tackled with by using simple tools:

A physical law “Snell’s law”3– which will not be discussed here – links the direction θ2 of a wave train on a depth h2 with the direction θ2 of the same wave on a given depth h1

$$\sin(\theta_2) = \frac{C_2 \sin(\theta_1)}{C_1} \qquad 10$$

and

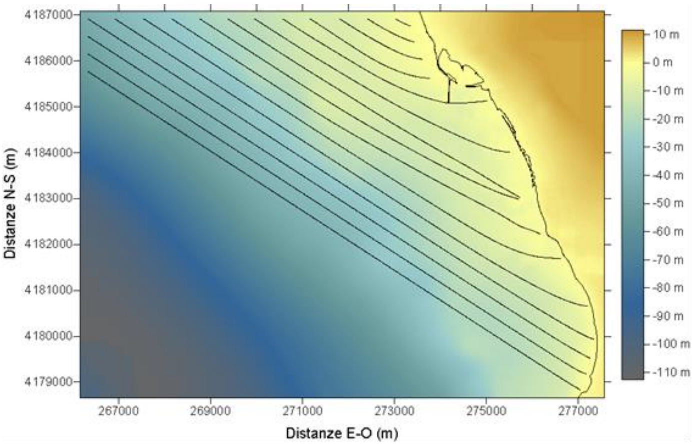
[3] Apparently, discovered by Ibn Sahl in the 10th Century. https://en.wikipedia.org/wiki/Ibn_Sahl

$$\mathrm{H}_2 = \mathrm{H}_1 \sqrt{\frac{cos(\theta_1)}{cos(\theta_2)}} \qquad 11$$

$$Kr = \sqrt{\frac{cos(\theta_1)}{cos(\theta_2)}} \qquad 11^*$$

(The directions θ are taken with respect to the perpendicular to the shoreline)

By making use of these formulas, the wave rays can be tracked ("ray tracing") from offshore to the coast: the calculations are cumbersome and can only be carried out with complex software systems



Calculation of refracted wave rays

If however some simpler condition are assumed, i.e. straight and parallel bathymetric lines, the procedure is simple enough:

Assume a wave system on deep water with the following parameters:

Ho T θ_0

We require H2 and θ2 at a depth H2

We use the formulas above (now the point "1" is the offshore position "o")

$$\mathrm{H}_2 = \mathrm{H}_0 Ks\, Kr$$

Where

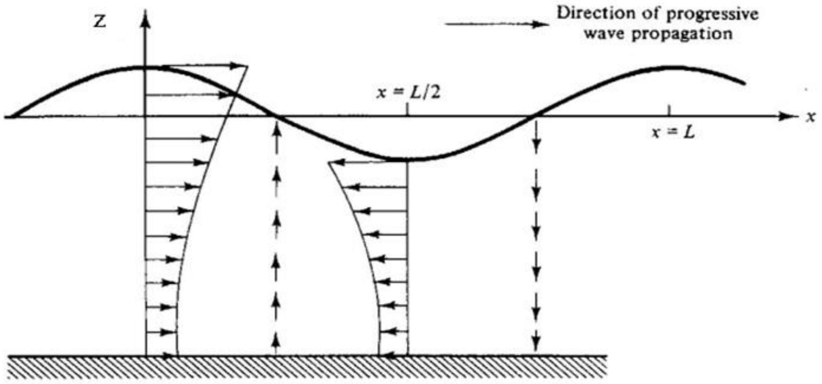
$$Ks = H / Ho = \sqrt{\frac{\mathrm{C_{g0}}}{\mathrm{Cg}}} = \sqrt{\frac{\mathrm{C_0}}{\mathrm{C}}\frac{0.5}{\mathrm{n}}}$$

And Kr is given by 11*

Exercise: compute and draw the diagrams of Vz and u, for different T, L, H

PARTICLE VELOCITY

Remember, the velocity Vx Vz of the water particles in NOT the velocity (aka celerity) of the wave system



Vx=u (Horizontal component) and
Vz (Horizontal component) are actually given by:

$$Vz(x,t) = \frac{H}{2}\sigma \sin(kx\text{-}\sigma t) = \frac{H\pi}{T}\sin(kx - \sigma t)\frac{\sinh k(h+z)}{\sinh(kh)} \qquad 1$$

$$u(x,z,t) = a\omega \frac{cosh\big(k(z+d)\big)}{sinh(kd)}\cos(kx-\omega t) \qquad 2$$

Formula 2 will be used in the follow to evaluate wave actions on the bottom.
In shallow waters, the formula simplifies- See exercises,

The formulas can be found on:
https://en.wikipedia.org/wiki/Airy_wave_theory

WAVE BREAKING ON SHALLOW WATERS

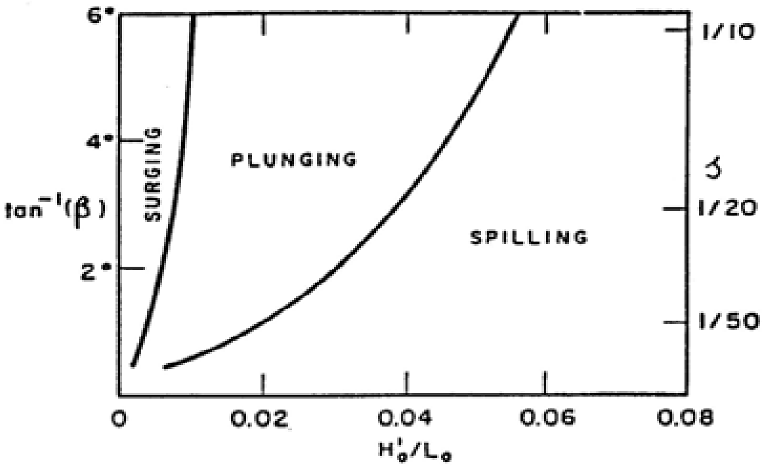
Waves are said to "break" when they lose they regular behaviour and turn into an irregular and turbulent motion; the breaking process can be easily identified from the intense formation of foam. Wave can break offshore (whitecapping), but we are here only interested in coastal processes.

The physical mechanisms which govern wave breaking are complex and hard to describe or compute; a common index is Iribarren number, which is basically the slope of the bottom tan(β) divided by the square roort of the offshore "wave slope" Ho/Lo

$$\xi_0 = \frac{\tan(\beta)}{\sqrt{(H_0 / L_0)}}$$

There are three main types of wave breaking: " spilling", "plunging", "surging" (other names and classea re possible), and they are linked to Irribarren's ξo according to the following table

Breaking type	?
spilling	<0.4
plunging	0.4-2.0
surging	>2.0



Regioni tipiche delle diverse figure di frangimento

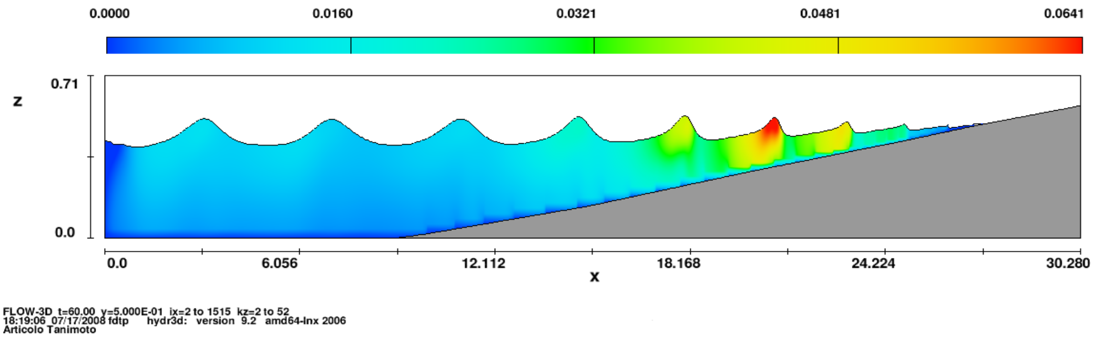
Spilling

ξo < 0,4 ; The behaviour is "dissipative" (Most of the wave energy is dissipated as the wave moves along the beach cross section, so that only a small percentage of the incoming energy is reflected)

It takes place mostly on mild slope coasts (beaches)

Spilling breaker

The wave shape during the braking is approximately symmetric around the crest. Most of the foam is generated on the front of the wave; its name derives from the spilling of foam. As the energy dissipates, the wave starts again, but with a smaller height; then it breaks again, and the process starts anew. The energy therefore is spilled from the wave path, hence another explanation for the name.

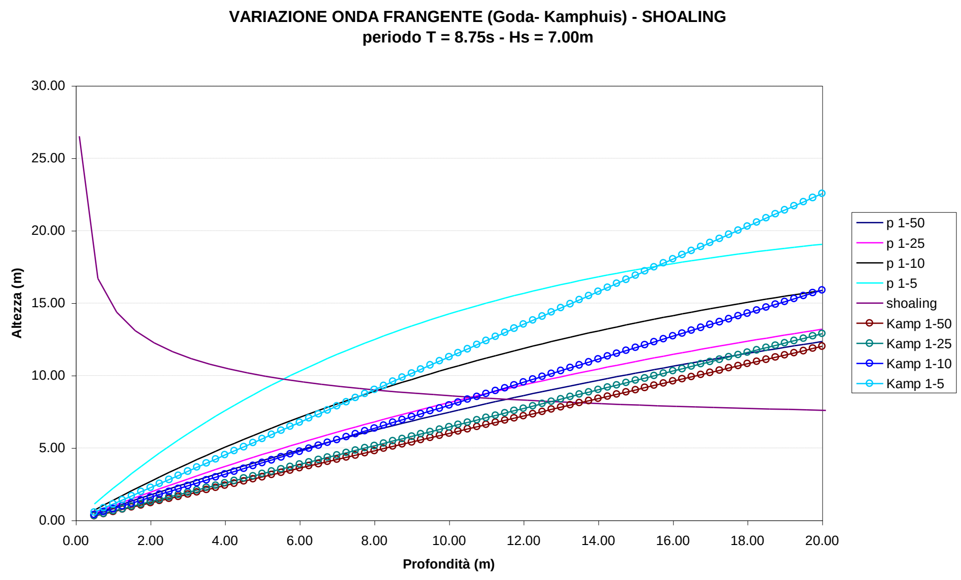


Spilling: the colours show the intensity of turbulence

Spilling breakers are particularly important in coastal management: beaches have a small slope and therefore most of the wave braking is of the spilling type (with some exceptions: tsunamis!) . It is therefore useful to examine with some care what happens when a spilling wave train approaches the coastline.
The depths at which spilling waves break is given by empirical rules, the simplest of which is the following:

$H_b=0.8\ h_b$ **

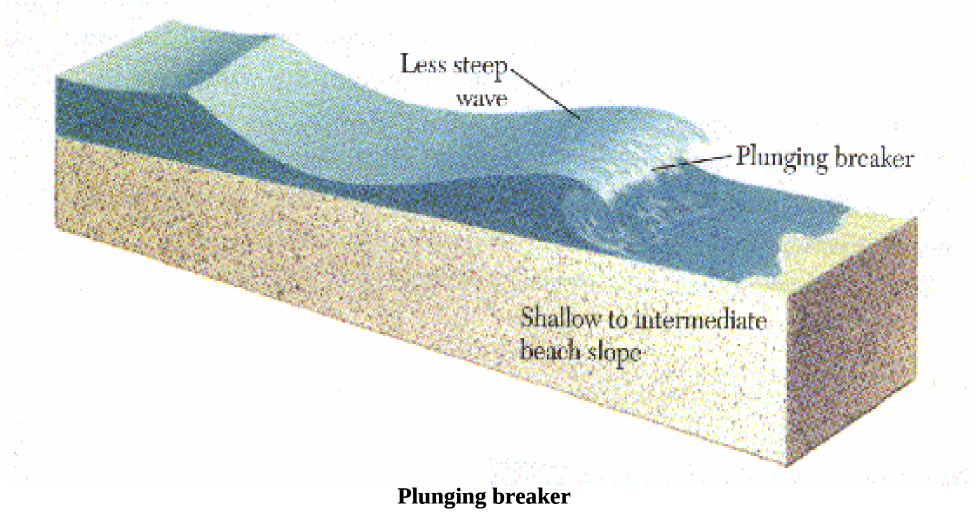
i.e. the wave breaks when it height Hb is equal to 0.8 the water depth hb. At that point most of the energy id dissipated (remember: the energy is give by the square of the wave height) and the wave movement starts again with a reduced H; as the bottom gets shallower and shallower, the waves will come again to a point where the equation ** is verified, so more energy is dissipated, and the wave will start again with a lower energy. And so on until all the energy is dissipated, and the wave height is negligible. This can be represented in the depth/height diagram by the strait line H=0.8 h which intersect the shoaling curve.



Plunging

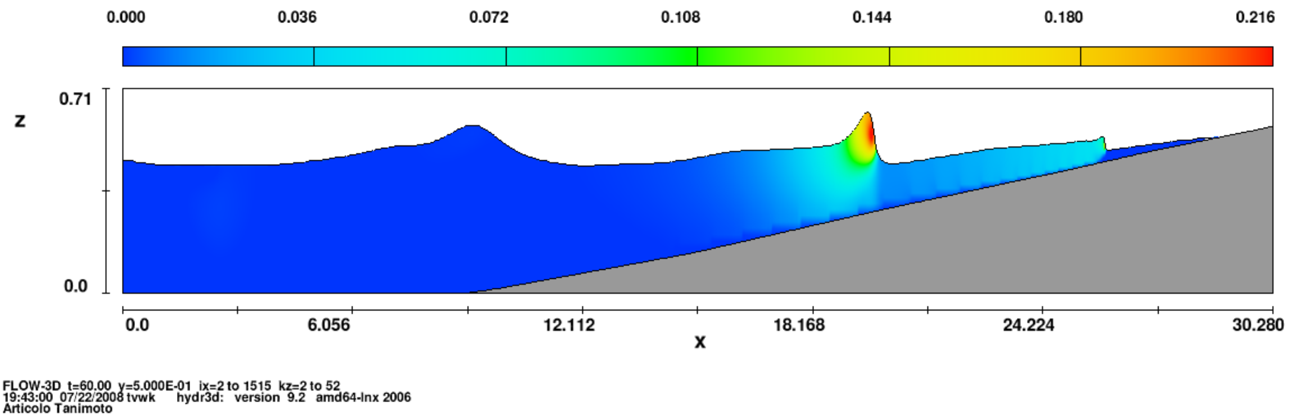
This breaker is (probably) a plunging wave

0,4 < ξo < 2 ; An intermediate behaviour from the point of view of energy dissipation and reflection.



Plunging breaker

The plunging breaker presents a non symmetric section around the crest; very often a jet is formed on the top, which eventually plunges (hence the name) into the main body.



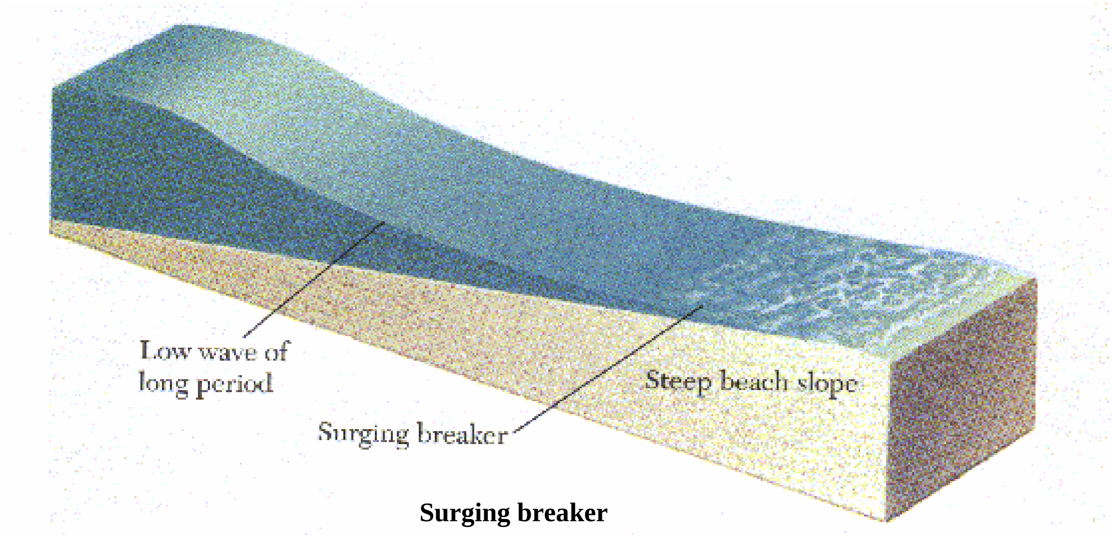
Plunging: the colours show the intensity of turbulence

Surging

ξo > 2 ; reflective behaviour (An important fraction of the energy is reflected)

Here is a , surging wave on a quasi vertical wall.

Surges are generated by the surging of water before the actual breaking; they usually take place near steep rocky cliffs or vertical breakwaters, but they can also happen on shallow beaches, ask long as the wave slope Ho/Lo is sl enough (see Irribarren Number)

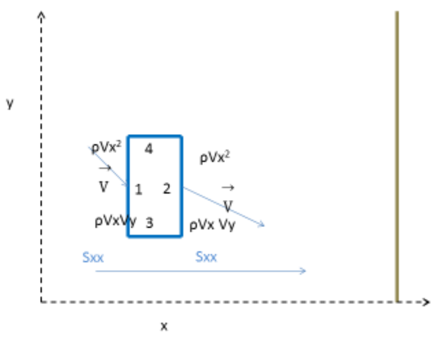


Surging breaker

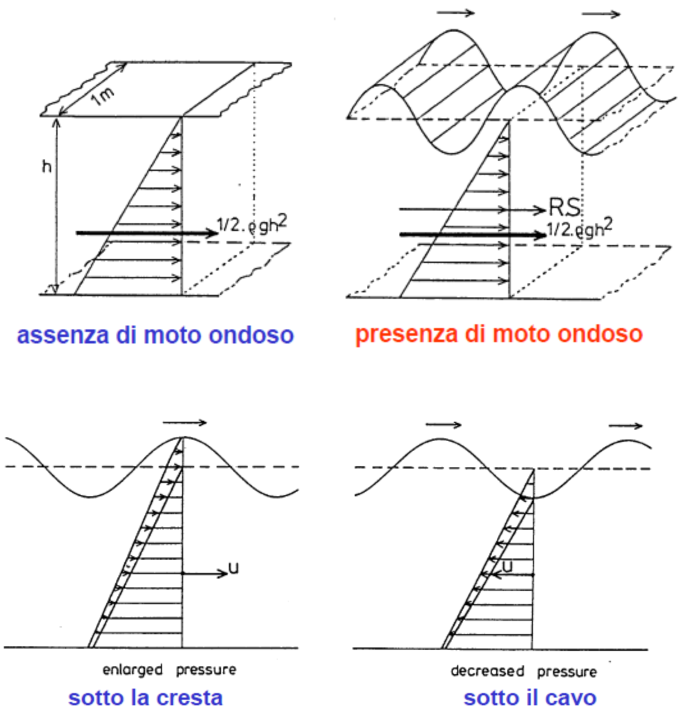
“RADIATION STRESSES”

Wave action induces a drag on the water mass in the nearby of the shoreline wheichy in turn least to the formation of a long-shore current

In order to explain such a mechanism, let’s consider a control volume, whose projection on the horizontal plane is shown in the figure below:



The momentum flux through the vertical surfaces is given by $\rho \vec{V}\ V_n$, where $\vec{V}$ is the orbital velcoity, and , Vn its normal component; this quantiyy varies with a sine time law as it governeb by Ary theory.



By averaging over time we obtain an additional term": the radiation stress which is a tensor in two dimensions, whose components are **Sxx, Sxy, Syx, Syy**.

If the wave conditions on the two opposite sides, i.e. planes 1 and 2 or 3 and 4, are identical, the so are the radiation stresses on the opposite sides of the water column, so no additional force is generated; if wave condition do vary, the a force si generated,

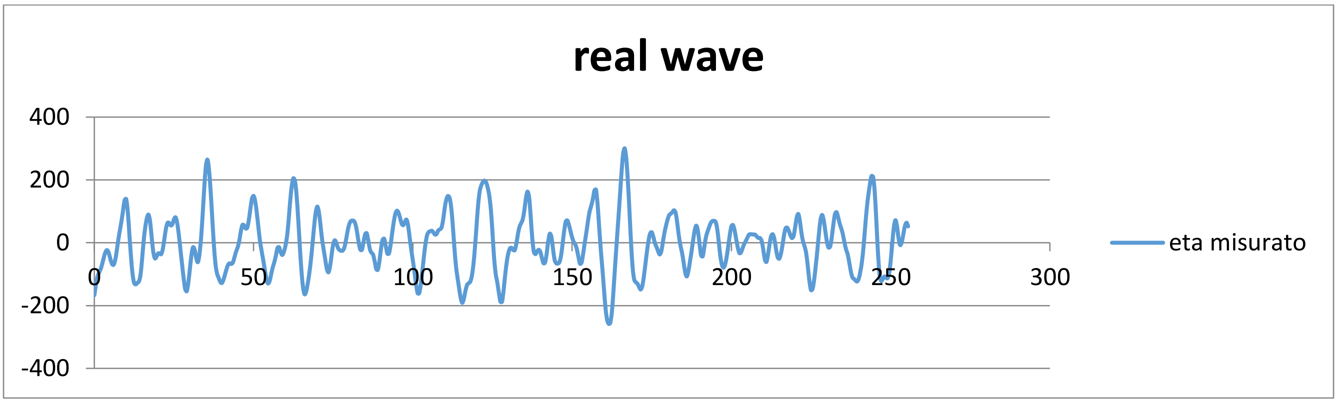
An useful result can be obtained by assuming parallel bathymetric lines , and a wave train approaching with an angle α to the perpendicular x to the coast.

$$S_{xy} = \mathrm{E} \cdot \mathrm{n} \cdot \cos(\alpha) . \sin(\alpha) = \mathrm{E} \cdot \mathrm{n} \cdot \cos(\alpha) . \sin(\alpha)$$

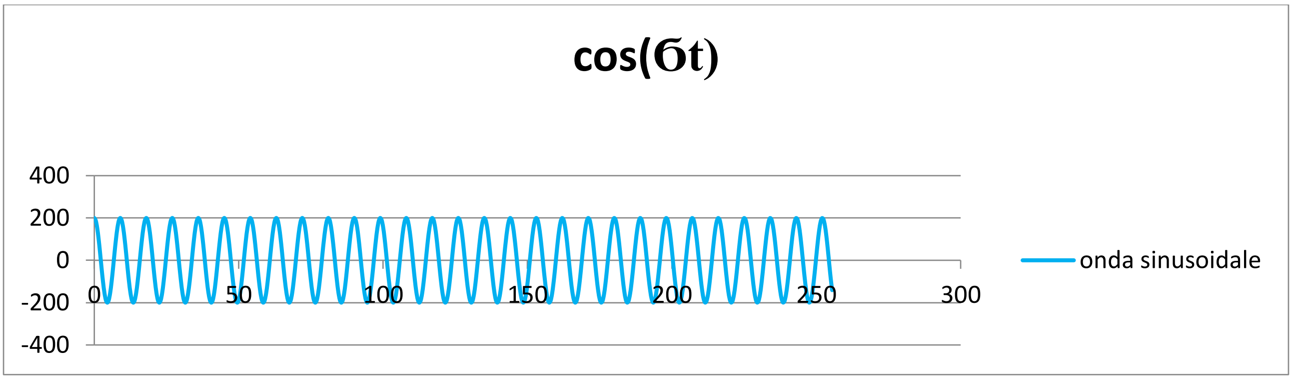
Remember that E= $\left(\frac{1}{8} \rho g H^2 \right)$

Real waves - sea states

If we measure water height at sea as function of time η(t) we get something like this:



Which is a far cry from what we have assumed to be a "wave":



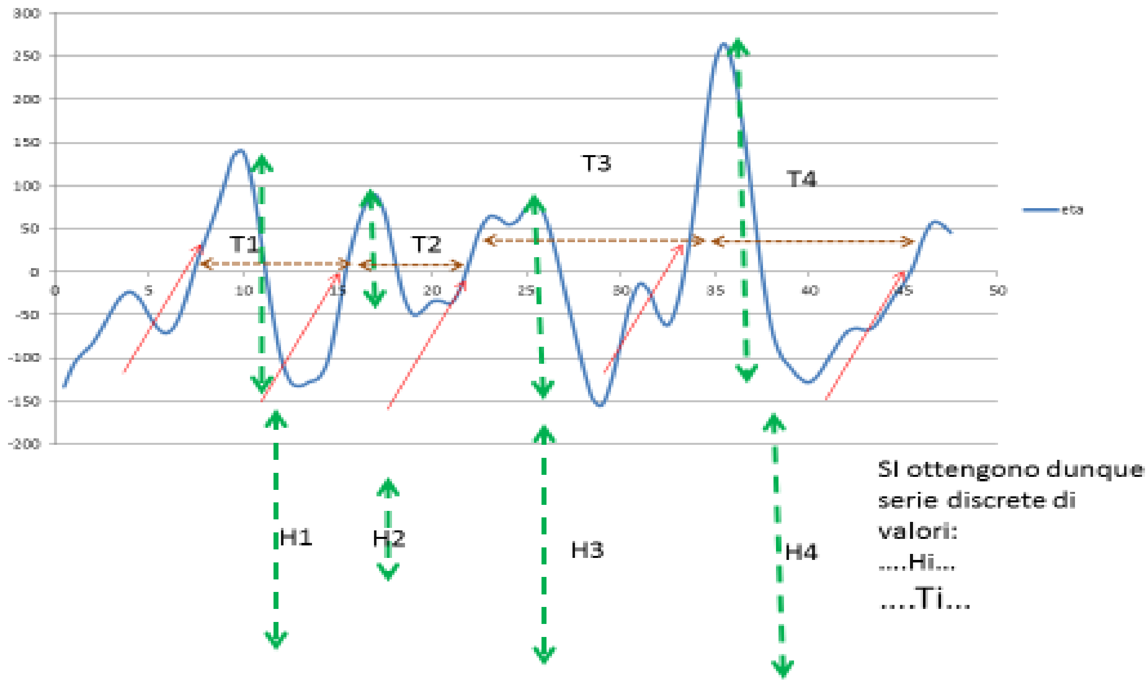
So we shall consider real "sea states" i.e. the water agitation for a given length of time (20' to a few hours). The water agitation in a sea states is chaotic, and it cannot be treated as a simple Airy wave as we have done so far; there are different ways of dealing with it, but here we shall only consider statistical approach. A sea state will therefore be characterized by its length, by the mean period of its waves, and by some kind of statistical parameter of its waves heights.

IMPORTANT: a sea state is made up of many waves, of different heights and periods. Its main parameters are:

Tm: mean wave period; Dir= Average direction of the waves; Ho (aka H1/3) "significant wave height"; Tm average wave period.

Time series of such parameters are measured by wavemeters, or computed by wavemodels; either way, long time series of waves are available for any points at sea. From such time series the statistical properties of "wave climate" on points offshore and along the coast will be derived during the exercises.

A sea state

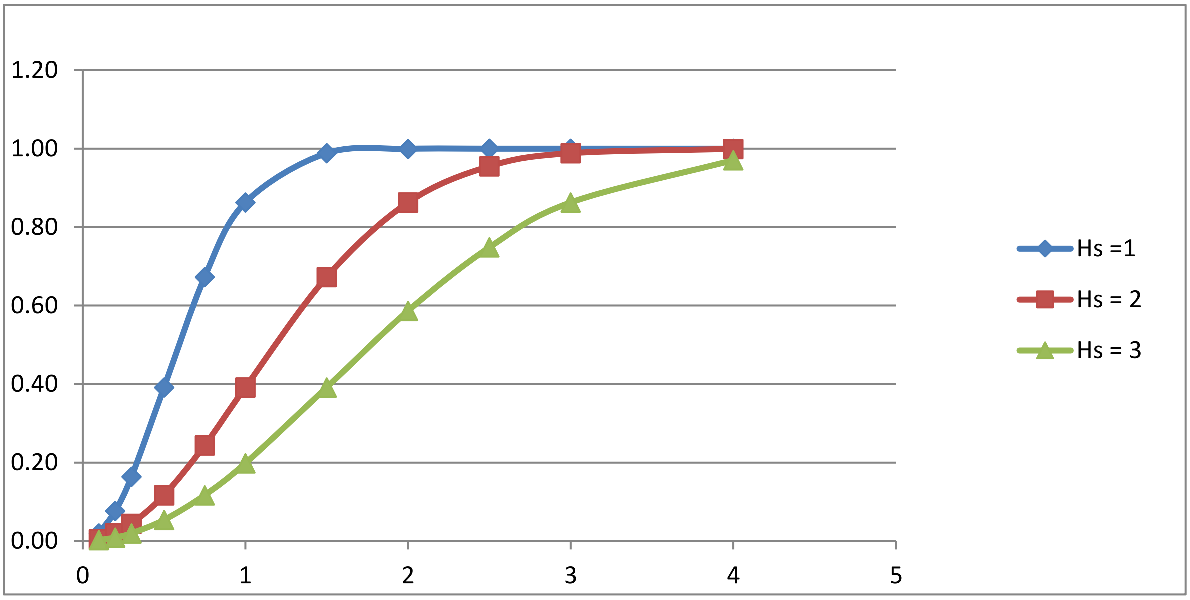


If we consider a sea state from this point of view, we have to deal with a number of single waves Hi, (i.e. no longer with a continuous sine wave). We treat them as random events and we consider they statistical behaviour.

The waves follow the "Rayleigh" frequency distribution p(H), and its cumulate p(H): single parameter distribution.

$$p(H) = \frac{2H}{H^2_{rms}} \exp\left(\frac{-H^2}{H_{rms}{}^2}\right)$$

$$P(H) = 1 - \exp\left(\frac{-H^2}{H_{rms}{}^2}\right)$$



This is the P(H) (Cumulate). *Exercise: plot p(H) (distribution)*

Hs is

$$H_{rms} = \left(\frac{1}{N} \sum_{i}^{N} H_i^{\,2} \right)^{1/2}$$

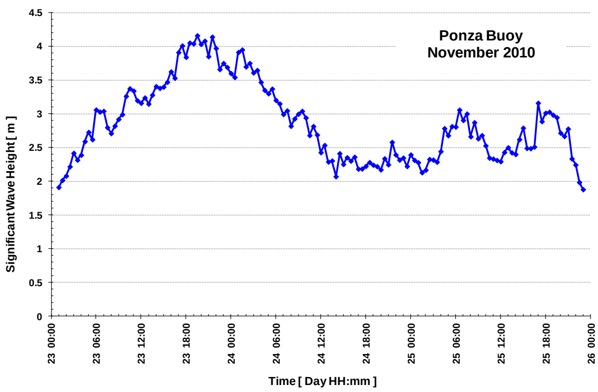
The intensity of a sea state is usually indicated by the so called “significant wave height”:

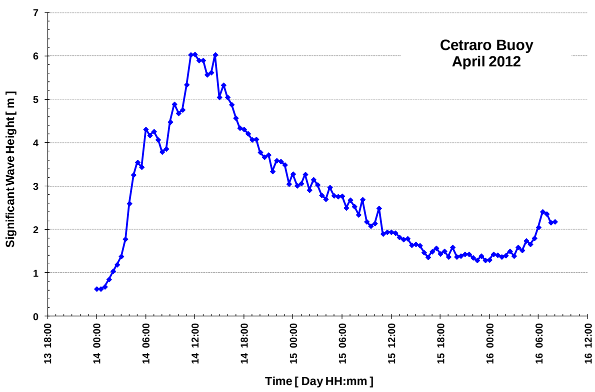
$$H_{1/3} = \sqrt{2} * H_{rms} \qquad \cong 1{,}41 H_{rms}$$

Another important parameters in a sea state is the “average wave period” Tm. Its definition is obvious.

The distribution of wave heights in is necessary in order to understand the behaviour of wave run up over a beach or a sea wall- an important aspect of coastal protection.

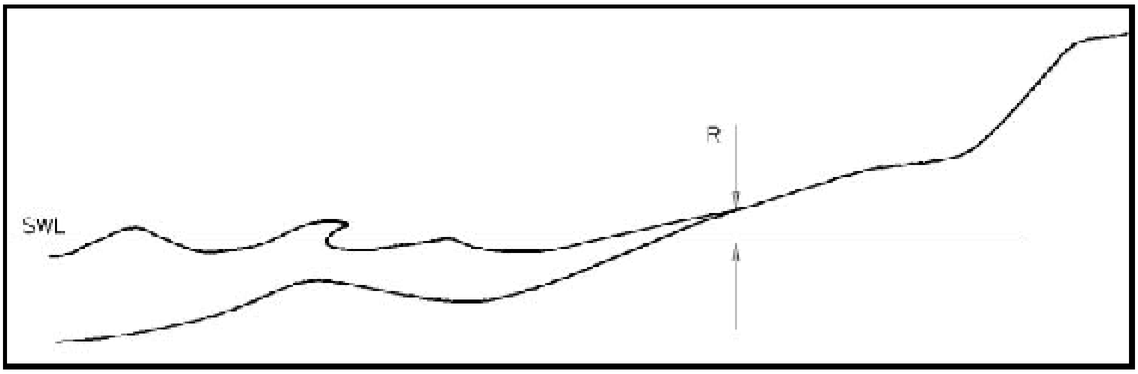
A succession of sea states, with particularly high values of Hs, is called a “sea storm”

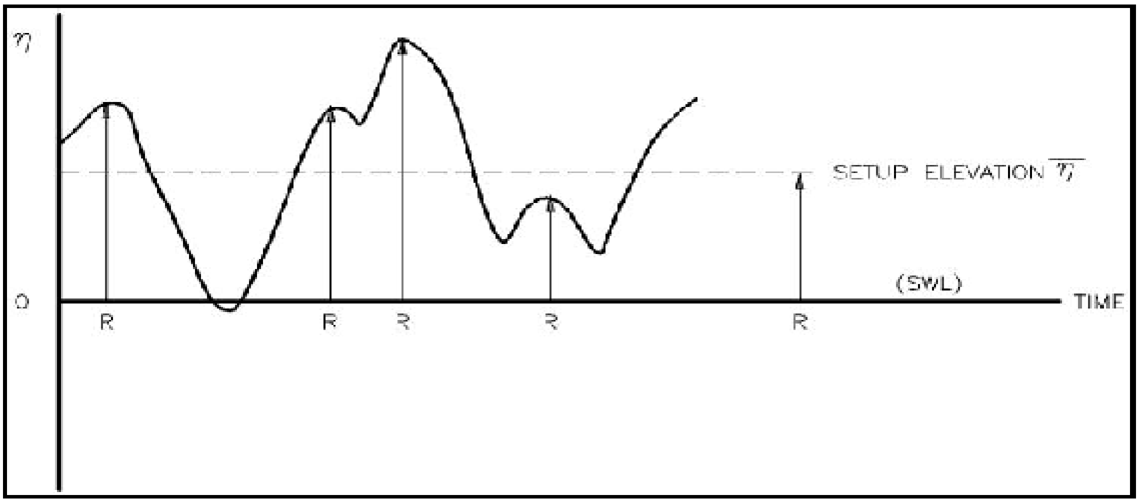




Wave run-up

In the breaker area part of the moment associated with the oscillating movement is sonverted into a forward and upwards translation of the water mass. This is the so called **run-up Ru,** defined as the elevation of the sea water height on a coast





The run up is a random quantity – since it is generated by a random succession of waves; it is also to be expected that its distribution should be similar to the offshore wave height distribution i.e. the Rayleigh function described above.

Run-up is the cause of coastal flooding, that can be dangerous for beaches and beach establishments and buildings[4],

It is therefore important that the distribution of extreme Run-up values should be known as a function of the offshore sea conditions. This is usually made through empirical formulas, such as for instance, Maze's: (No need to learn the numerical constants, just the structure of the formulas)

[4] Italian laws, for instance, prescribe that before a beach establishment is authorized, a risk analysis must be carried out

$$\frac{R_{2\%}}{H_0} = 1.86 \cdot \xi^{0.71} \qquad \frac{R_{1/10}}{H_0} = 1.70 \cdot \xi^{0.71} \qquad \frac{R_{1/3}}{H_0} = 1.38 \cdot \xi^{0.70}$$

$$\frac{R_{medio}}{H_0} = 0.88 \cdot \xi^{0.69}$$

Where

R2%= the run-up value of the 2% highest waves

R1/10 = the run-up value of the 10% highest waves

R1/3 = the run-up value of the 1/3 highest waves 1/3

Rmedio= the average run-up

$$\xi_0 = \frac{\tan(\beta)}{\sqrt{(H_0 / L_0)}}$$ is obviously Irribarren offshore number

Wave Data

There are two main sources of wave data: wavemeters and wave models.

The traditional - and until recently, the prevalent - source of data is the historical wavemeter record: Wave measurement is usually based on a floating anchored buoys which measure water height and, after complex elaboration, produce time series of sea state data.

The following picture (from the Italian Environmental Agency-ISPRA handbook) shows the Italian Wavemeter Network (RON). Even if RON is not working now, there should be enough data to provide a reliable wave climate for each locations.

The position of the wavemeter buoys is reported on pag 6 of http://www.isprambiente.gov.it/files/atlante-coste/capitolo1.pdf

At present, one of the few functioning public wavemeter is the buoy run

by the Salerno Provincial Authority (http://buoy.ageotec.com/salerno/), under the scientific supervision of CUGRI

In the last few years the availability of long time series of computed wave field, produced over large computational grids by Weather Models and Wave Models chains (WaN - WaM), and corrected by assimilating measure wave data. has completely changed the picture. Procedures based on such synthetic data are becoming more and more common to the point that they will probably become dominant in the next few years. Global as well as of local area models are run mostly by national and international weather services, such as for instance ECMWF (European Centre for Medium-Range Weather Forecasts), NOAA (National Oceanic and Atmospheric Administration), UKMO (United Kingdom Meteorological Office) etc. The number of data providers is increasing, with private companies joining state and international organizations in supplying – at a price - both historical time series and statistical analyses for any site in the world.

.No extensive review of the literature is here possible, or indeed useful, but the interested reader may find the relevant information on the European Centre for Medium range Weather Forecasts (ECMWF) library, which is constantly updated,

The quality of the results depends on a wide numbers of factors. Even the concept itself of WeM+WaM varies enormously: e.g., the meteorological part WeM may be either global or regional with possibly various level of nesting within the global framework; the computational grid of the wave model WaM, while obviously following the WeM structure, may be further divided into finer resolution sub-areas; assimilation of in situ data, calibration and validation may be carried out in various ways.

Spatial grid resolution of both WaN and WaM has been improving for many years due to the advances in technology, to the point that modern local models provide a resolution of a few miles, which can realistically be considered the maximum practically attainable.

Model data are available among others by:

http://www.ecmwf.int/en/forecasts/datasets/set-ii e su

and from NOAA

http://polar.ncep.noaa.gov/waves/ensemble/

There are many other sources: e.g. the Italian Air Force

http://www.meteoam.it/prodotti_grafici/statoMareVento10metri

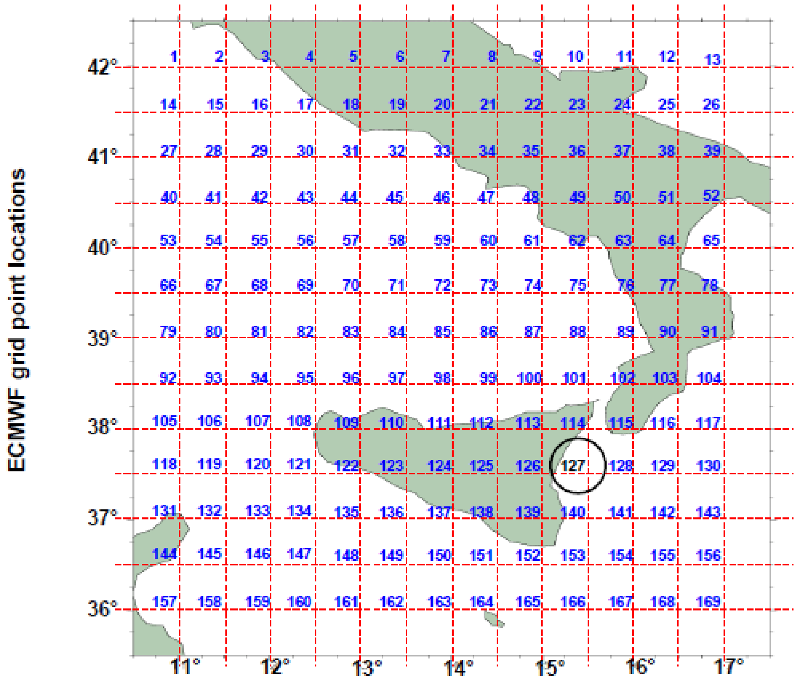
A similar system is operational at ISPRA

http://www.isprambiente.gov.it/pre_mare/coastal_system/maps/2015111100/med/mediterraneo.html

And another one at the CCMMMA (Parthenope University, Naples)

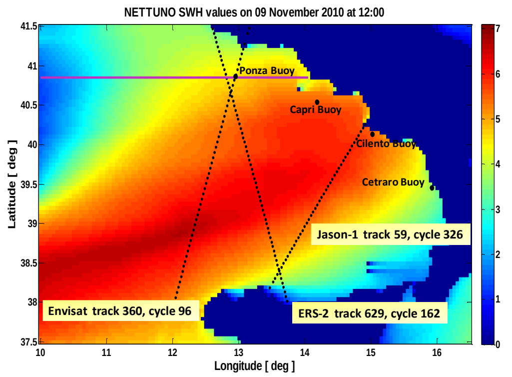
http://ccmmma.uniparthenope.it/sais

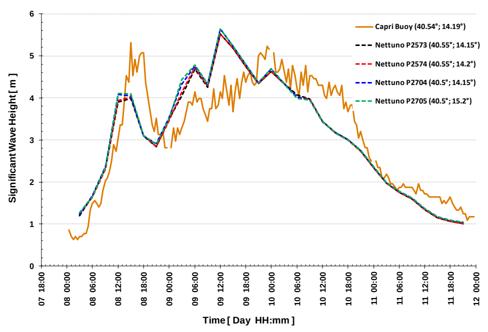
This last is particularly interesting since it provides real time warning data for beach safety. The system is currently being calibrated.

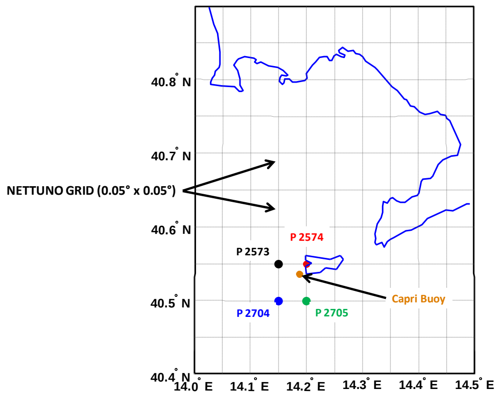


This is a typical "grid".

In the following, a comparison between model and experimental data:

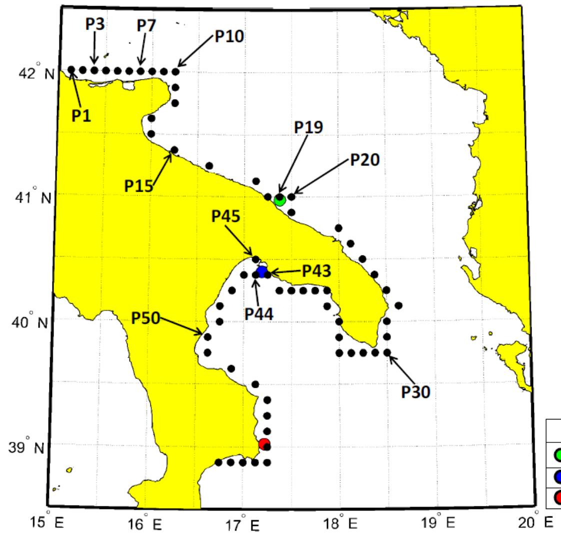






Top-left: CNMCA (Italian Air Force Meteo Office, Courtesy LT. Col Torrisi) Nettuno SWH simulation on 09 November at 12:00; Top-right: comparison between Capri buoy and Nettuno model grid points; bottom: location of buoy and grid points Buoy data from Civil Protection Dept., Campania Region, Courtesy Ing Biafora)

Simulated data series are an important basis to investigate on wave climate and therefore to evaluate coastal hazards. The following figure shows the ECMWF grid points around the coast of Apulia, Calabria and Lucania.



Punto	Lat.	Lon.
1	42	15.125
2	42	15.25
3	42	15.375
4	42	15.5
5	42	15.625
6	42	15.75
7	42	15.875
8	42	16
9	42	16.125
10	42	16.25

Punto	Lat.	Lon.
19	41	17.375
20	41	17.5

Punto	Lat.	Lon.
43	40.375	17.25
44	40.375	17.125

Boe	Lat.	Lon.
Monopoli	40.975	17.378
Taranto	40.4	17.19
Crotone	39.024	17.22

Data are collected at various time intervals: from 6hrs (ECMWF) down to 30' or 20' (wavemeters).

Long wave records are usually kept and distributed as files which report the important parameters of each sea state: Hs, Tm, DIRection. There is much more information, but it is not needed for the analyses we shall be carrying out in this course

In order to estimate the energy flux for a se state we must transform the

$$\overline{F} = E \cdot C_g = E \cdot C \cdot n$$

(In watt, for a one meter wide strip)

Considering C and n in deep water:

$$\overline{F} = E \cdot C \cdot n = \frac{g.\rho}{8} \frac{gT}{2\pi} H^2 \bullet 0{,}5 = \frac{g.\rho}{32} \frac{gT}{\pi} H^2$$

Dealing with a succession of random waves, since the wave height appears with its square, it makes sense to use the mean square average H_{rms}, hence:

$$\overline{F}_{random} = \frac{g.\rho}{32} \frac{gT}{\pi} H_{rms}^{\ 2}$$

By making use of $H_{rms} = H_{1/3} / \sqrt{2}$:

$$\overline{F} = \frac{g.\rho}{64} \frac{gT}{\pi} H_{1/3}^{\ 2}$$

(No need to memorise; but you must be able to reconstruct it)
For T , assume Tm. The result is still in Watt/m