**A quick (refresher) course on sea waves**

Parts of the text marked in blue are derivations or analytical details that are useless at this stage. They are there only to provide continuity to the discussion.

Parts of the text marked in yellow are optional: you can skip them, but you might find them useful in the future

*Paragraphs in italics suggest exercises to be carried out in the classroom or as homework*



Vincent van Gogh, View of the Sea, Scheveningen, 1882 https://www.vangoghmuseum.nl/en/collection/s0416M1990

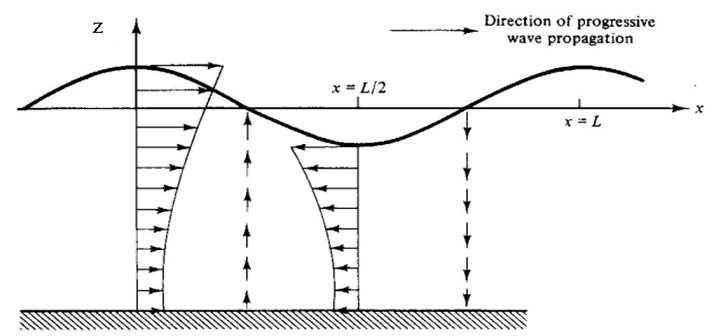
**Airy / Stokes I**

A simplified model of water waves is provided by Airy / Stokes I  theory



Also sometime called “ a sine wave” since the instantaneous water height η(x,t) is a sine function of space x and time t[[1]](#footnote-1).

k:wave number; k= 2π/L; L: wavelength ; σ frequency; σ = 2π/T; T :period



(from Dean e Dalrymple, 1991)

It can be shown that the whole “wave train” moves with velocity C=L/T.

We therefore define

C=L/T “Wave speed” (also: “ celerity”)

Please note that this is the velocity of the wave system, NOT the velocity of water particles- which will be considered in the following

A very important remark: σ=2π/T e di k=2π/L (and therefore T and L ) are bound to each other according to the equation

 **3 \***

(h being the water depth)

This very important formula (“**dispersion equation”)** can be written in many ways: e.g.

 3\*\*

Or , else:

 3\*\*\*

As stated above a sine wave train will move over a distance L in time T

From the previous, we also have the following

3\*\*\*\*

**(you don’t have to learn all these formulas!: just one form, and then you can derive the others)**



**It useful and important to remember the asymptotic val**ues for deep water and shallow water:

**C0 is the wave speed on deep water (h>>L) and is given by**

**4**

**Also:**

**While in shallow water (h<L/10)**

**4\*\***

*Why is it called “dispersion” equation?*



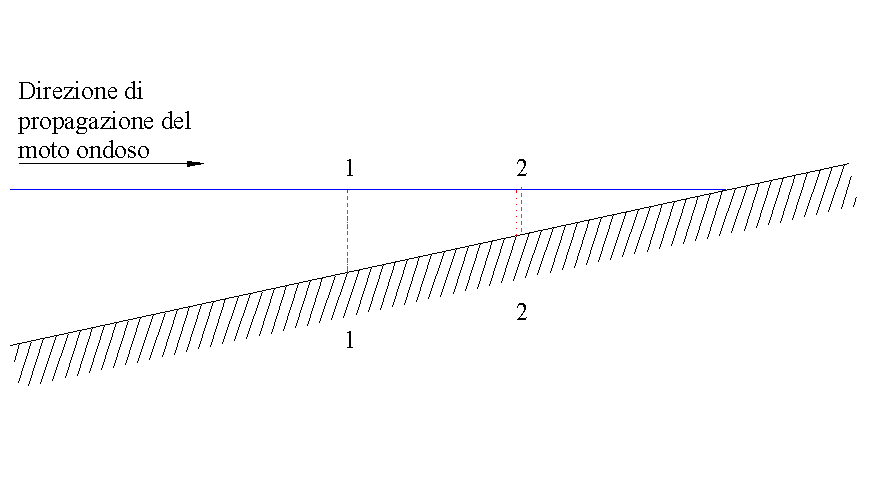
*Are the waves dispersive in deep water? And in Shallow water?*



**Shoaling**

As a wave train approaches the coast, it undergoes some transformation due to the effect of the water depth h.

We shall assume a mild slope of the bottom (basically: a beach), with a linear and parallel bathymetry.





as the wave moves from off-shore to shore (1 towards 2 in the picture) there is no change of the period T while L and C change according to the **dispersion equation**:

3\*\*\*\*

And

Remember: k= 2

There is a technical difficulty in dealing with the equations above: they cannot be solved for L since this unknown appears on both sides of the equation. For our purposes it suffices to know that there are various numerical approximations: One of them is Hunt’s formula, which is already coded in the EXCEL files provided in the exercises.

*We shall only apply this equation for the asymptotic case of Deep and Shallow Water*

Also the height H of a wave depends on the local depths h;

**Hs= Ho Ks** 6

Ks is the shoaling coefficient:

(  **7** NO NEED TO LEARN BY HEART but **you must** be able to use it in the applications)

Cg is called “Group Velocity and is given by Cg=n C;

And n is given by

 **8**

(NO NEED TO LEARN BY HEART but you must be able to use it in the applications for the asymptotic case : deep water n=0.5 and shallow water n=1 )

Here too the “asymptotic” solutions for deep water (L>h) and shallow water (L<<h) are very useful, and easy to remember

in deep water

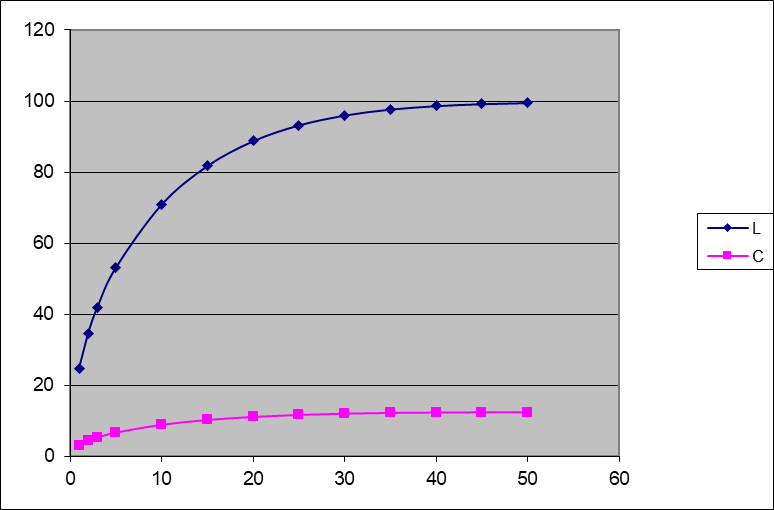
in shallow water

Ks links the wave height H at the local depth to offshore (deep water ).

Hence 7 becomes:

*~~Compute C , L , Ks on shallow water)- See exercises~~*

The following diagrams show the behaviour of C, L and Ks as a function of depth for a wave train that approaches the coast



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shoaling coefficient K as a function of depth h for various values of wave period T

**REFRACTION**

If an Airy/Stokes I sine wave train moves over the sea surface, it can be visualized as follows.

The parallel lines represent points of equal phases e.g. the crests, or the troughs

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-

H/2

H/2

H/2

H/2

H/2

H/2

They are also called wave fronts. Perpendiculars to the wave fronts are called “rays” and the vectors along the rays are called “propagation vectors” (green arrow on the picture above)

If the wave fronts are rectilinear, the theory and the formulas described above apply without any change. The rays are straight lines, and the propagation vectors are all aligned.

The wave fronts can also be curves (for reasons that will be explained in the following); if the curvature is not too high (local curvature radius >>L) the same results apply along the rays (green thick lines )



It is therefore acceptable to assume that along the propagation (rays) the wave behaves nearly as the one-dimensional wave described above

When the waves move over a varying bathymetry (depth profile) the wave celerity changes as a function of depth h; but h(x,y) is generally different along the wave front, so it will change its shape and the rays will rotate. This effect is called “rifraction”

A problem which often arises in connection with the coastal sand regime, is how to calculate the angle of arrival of a wave train on the beach, once its direction offshore is known

However, conceptually at least, the problem can be tackled with by using simple tools:

A physical law “**Snell’s law”[[2]](#footnote-2)–** which will not be discussed here –

links the direction θ2 of a wave train at depth h2 with the direction θ1 of the same wave at a given depth h1

**10**

(Important)

and

=Hi Ks Kr **11**

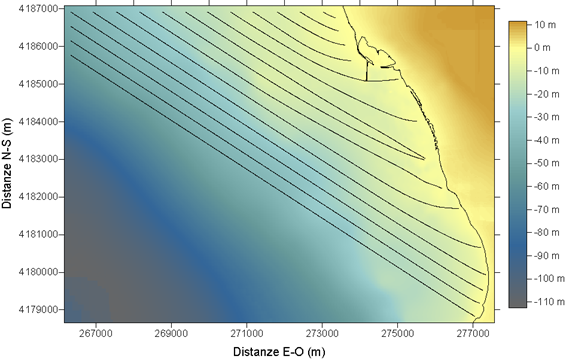
(The directions θ are taken with respect to the perpendicular to the shoreline)

Where:

11\*

**NO NEED TO LEARN BY HEART** but you must be able to use it in the applications, from deep to shallow water

By making use of these formulas, the wave rays can be tracked (“ray tracing”) from offshore to the coast: the calculations are cumbersome and can only be carried out with complex software systems



Calculation of refracted wave rays

If however some simpler condition are assumed, i.e. straight and parallel bathymetric lines, the procedure is simple enough:

A*ssume a wave system on deep water with the following parameters:*

*Ho T θ0*

We require H2 and θ2 at a depth h2

We use the formulas above (now the point “1” is the offshore (=deep water) position “o”)

Where

 n=1

**(tra acque profonde e acque basse n=1)**

And Kr is given by 11\*

11\*

**WAVE BREAKING ON SHALLOW WATERS**

Waves are said to “break” when they lose they regular behaviour and turn into an irregular and turbulent motion; the breaking process can be easily identified from the intense formation of foam. Wave can break offshore (whitecapping), but we are here only interested in coastal processes.

The physical mechanisms which govern wave breaking are complex and hard to describe or compute; a common index is **Iribarren number**, which is basically the slope of the bottom tan(β) divided by the square roort of the offshore “wave slope” Ho/Lo

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There are three main types of wave breaking: “ spilling”, “plunging”, “surging” (other names and classea re possible), and they are linked to Irribarren’s ξo according to the following table

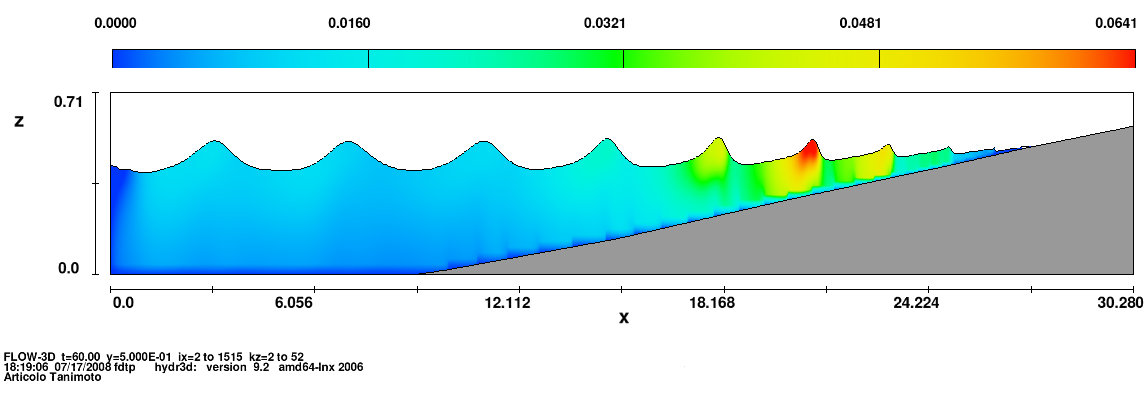
|  |  |
| --- | --- |
| Breaking type |  |
| spilling | <0.4 |
| plunging | 0.4-2.0 |
| surging | >2.0 |
|  |  |

**Spilling**

**ξo <  0,4** ; The behaviour is "dissipative" (Most of the wave energy is dissipated as the wave moves along the beach cross section, so that only a small percentage of the incoming energy is reflected)

It takes place mostly on mild slope coasts (beaches)

The wave shape during the braking is approximately symmetric around the crest. Most of the foam is generated on the front of the wave; its name derives from the spilling of foam. As the energy dissipates, the wave starts again, but with a smaller height; then it breaks again, and the process starts anew. The energy therefore is spilled from the wave path, hence another explanation for the name.



**Spilling:** the colours show the intensity of turbulence

Spilling breakers are particularly important in coastal management: beaches have a small slope and therefore most of the wave braking is of the spilling type (with some exceptions: tsunamis!) . It is therefore useful to examine with some care what happens when a spilling wave train approaches the coastline.

The depths at which spilling waves break is given by empirical rules, the simplest of which is the following:

***Hb=0.8 hb*** \*\*

i.e. the wave breaks when it height Hb is equal to 0.8 the water depth hb. At that point most of the energy is dissipated (remember: the energy is given by the square of the wave height) and the wave movement starts again with a reduced H; as the bottom gets shallower and shallower, the waves will come again to a point where the equation \*\* is verified, so more energy is dissipated, and the wave will start again with a lower energy. And so on until all the energy is dissipated, and the wave height is negligible. This can be represented in the depth/height diagram by the strait line H=0.8 h which intersect the shoaling curve.



**Plunging**

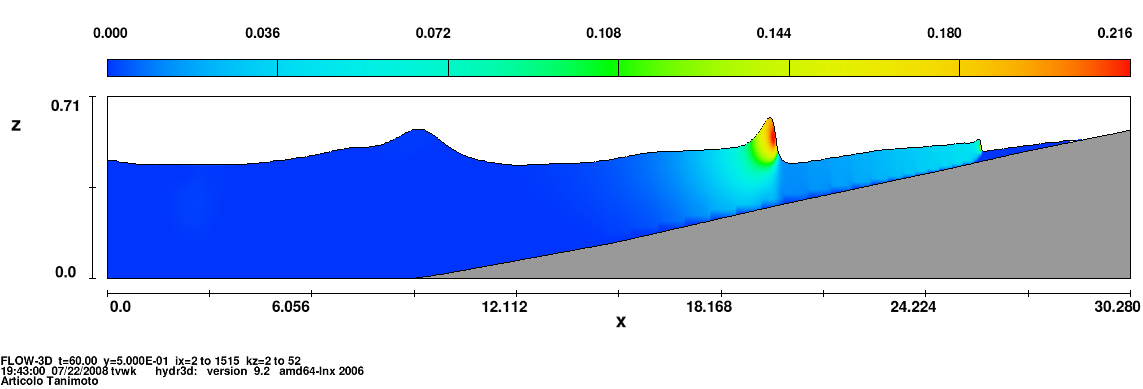


This breaker is (probably) a plunging wave

**0,4 < ξo < 2** ; An intermediate behaviour from the point of view of energy dissipation and reflection.



The plunging breaker presents a non symmetric section around the crest; very often a jet is formed on the top, which eventually plunges (hence the name) into the main body.



Plunging: the colours show the intensity of turbulence

**Surging**

ξo >  2 ; reflective behaviour (An important fraction of the energy is reflected)



Here is a , surging wave on a quasi vertical wall.

Surges are generated by the surging of water before the actual breaking; they usually take place near steep rocky cliffs or vertical breakwaters, but they can also happen on shallow beaches, ask long as the wave slope Ho/Lo is sl enough (see Irribarren Number)

1. Obviously a sine function is simply a shifted cosine [↑](#footnote-ref-1)
2. Apparently, discovered by Ibn Sahl in the 10th Century. https://en.wikipedia.org/wiki/Ibn\_Sahl [↑](#footnote-ref-2)